Pearson Edexcel Level 3

GCE Mathematics

Advanced

Paper 2: Pure Mathematics

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Time: 2 hours	Paper Reference(s)
	9MA0/02

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 14 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Answer ALL questions.

1. The figure 1 shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{ax+4}{x-b}$, $x > 2$

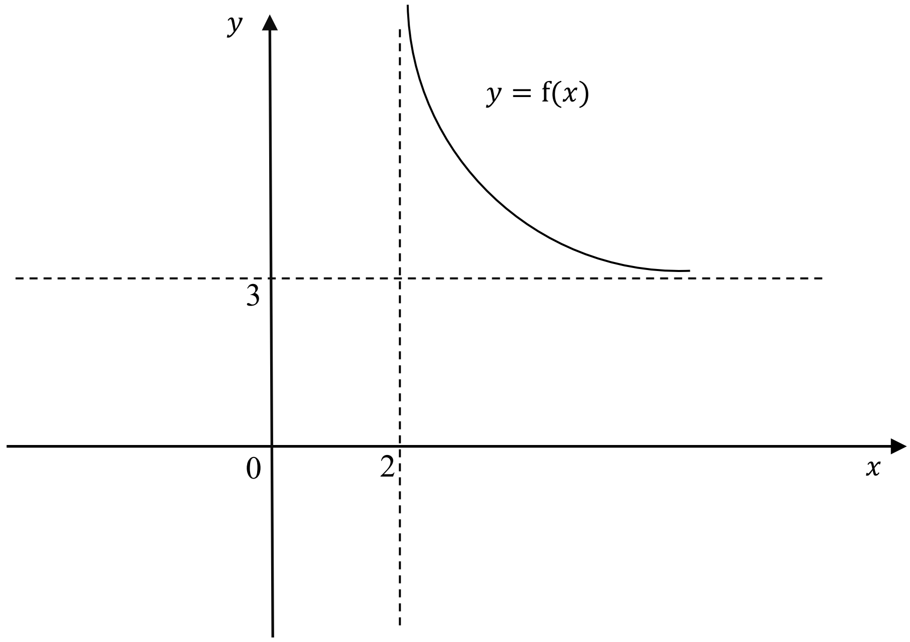

Figure 1

a. State the values of a and b.

(2)

b. State the range of f.

(1)

c. Find $\mathrm{f}^{-1}(x)$, stating its domain.

(3)

(Total for Question 1 is 6 marks)

2. Relative to a fixed origin O,

the point A has position vector $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

the point B has position vector $(\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$

and the point C has position vector $(-\mathbf{i} + \mathbf{j} + a\mathbf{k})$, where a is a constant and $a > 0$.

Given that $|\overrightarrow{BC}| = \sqrt{41}$

a. show that $a = 2$.

(3)

D is the point such that $ABCD$ forms a parallelogram.

b. Find the position vector of D.

(2)

(Total for Question 3 is 5 marks)

3. a. "If p and q are irrational numbers, where $p \neq q, q \neq 0$, then $\frac{p}{q}$ is also irrational."

Disprove this statement by means of a counter example.

(2)

b. (i) Sketch the graph of $y = |x| - 2$.

(ii) Explain why $|x - 2| \geq |x| - 2$ for all real values of x.

(3)

(Total for Question 3 is 5 marks)

4. (a) Show that $\sum_{r=1}^{20}(2^{r-1} - 3 - 4r) = 1047675$

(4)

(b) A sequence has nth term $u_n = \sin(90n°)\ n \geq 1$

(i) Find the order of the sequence.

(1)

(ii) Find $\sum_{r=1}^{222} u_r$

(2)

(Total for Question 4 is 7 marks)

5. $\mathrm{f}(x) = \frac{1}{3}x^3 - 4x - 2$

a. Show that the equation $\mathrm{f}(x) = 0$ can be written in the form $x = \pm\sqrt{a + \frac{b}{x}}$, and state the values of the integers a and b.

(2)

$\mathrm{f}(x) = 0$ has one positive root, α.

The iterative formula $x_{n+1} = \sqrt{a + \frac{b}{x_n}}$, $x_0 = 4$ is used to find an approximation value for α .

b. Calculate the values of x_1, x_2, x_3 and x_4 to 4 decimal places.

(2)

c. Explain why for this question, the Newton-Raphson method cannot be used with $x_1 = 2$.

(1)

(Total for Question 5 is 5 marks)

6. $\quad \mathrm{f}(x) = 2x^3 + 3x^2 - 1$

a. (i) Show that $(2x - 1)$ is a factor of $\mathrm{f}(x)$.

(ii) Express $\mathrm{f}(x)$ in the form $(2x - 1)(x + a)^2$ where a is an integer.

(4)

Using the answer to part a) (ii)

b. show that the equation $2p^6 + 3p^4 - 1$ has exactly two real solutions and state the values of these roots.

(2)

c. deduce the number of real solutions, for $5\pi \leq \theta \leq 8\pi$, to the equation

$$2\cos^3\theta + 3\cos^2\theta - 1 = 0$$

(1)

(Total for Question 6 is 7 marks)

7. (i) Solve $0 \leq \theta \leq 180^0$, the equation

$$4\cos\theta = \sqrt{3}\ \operatorname{cosec}\theta$$

(4)

(ii) Solve, for $0 \leq x \leq 2\pi$, the equation

$$\cos x - \sqrt{3}\sin x = \sqrt{3}$$

(5)

(Total for Question 7 is 9 marks)

8.

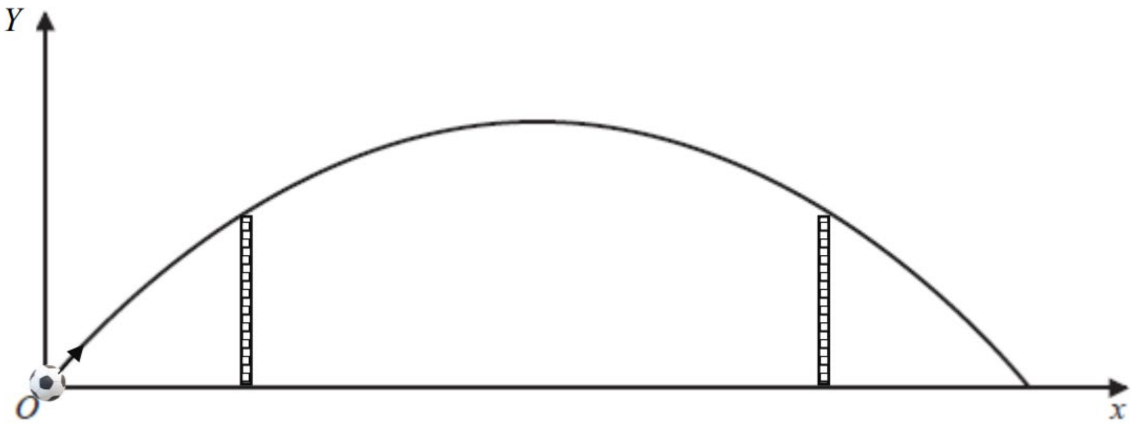

Figure 2

In a competition, competitors are going to kick a ball over the barrier walls. The height of the barrier walls are each 9 metres high and 50 cm wide and stand on horizontal ground.

The figure 2 is a graph showing the motion of a ball.

The ball reaches a maximum height of 12 metres and hits the ground at a point 80 metres from where its kicked.

a. Find a quadratic equation linking Y with x that models this situation.

(3)

The ball pass over the barrier walls.

b. Use your equation to deduce that the ball should be placed about 20 m from the first barrier wall.

(3)

c. Give one limitation of the model.

(1)

(Total for Question 8 is 7 marks)

9. Given that x is measured in radians, prove, from the first principles, that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0, \frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$.

(5)

(Total for Question 9 is 5 marks)

10. Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(12x+9)y^{\frac{1}{3}}}{x}$$

Giving your answer in the form $y^2 = f(x)$.

(Total for Question 10 is 6 marks)

11. $\frac{-6x^2+24x-9}{(x-2)(1-3x)} \equiv A + \frac{B}{x-2} + \frac{C}{1-3x}$

a. Find the values of the constants A, B and C.

(4)

b. Using part (a), find $f'(x)$.

(2)

c. Prove that $f(x)$ is an increasing function.

(1)

(Total for Question 11 is 7 marks)

12. a. Prove that

$$\frac{\sec^2 x - 1}{\sec^2 x} \equiv \sin^2 x$$

(3)

b. Hence solve, for $-360^0 < x < 360^0$, the equation

$$\frac{\sec^2 x - 1}{\sec^2 x} = \frac{\cos 2x}{2}$$

(4)

(Total for Question 12 is 7 marks)

13. a. Find $\int \ln x \ dx$

(4)

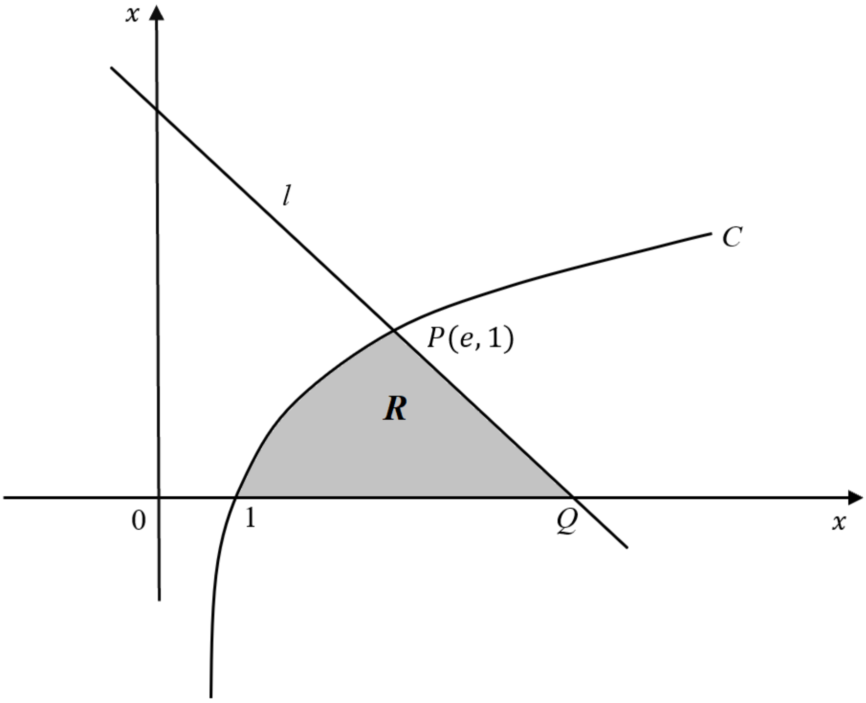

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = \ln x, \quad x > 0$$

The point P lies on C and has coordinate $(e, 1)$.

The line l is a normal to C at P. The line l cuts the x-axis at the point Q.

b. Find the exact value of the x-coordinate of Q.

(3)

The finite region **R**, shown shaded in figure 3, is bounded by the curve, the line l and the x-axis.

c. Find the exact area of **R**.

(3)

(Total for Question 13 is 10 marks)

14. A population of ants being studied on an island. The number of ants, P, in the population, is modelled by the equation.

$$P = \frac{900ke^{0.2t}}{1+ke^{0.2t}}, \text{ where } k \text{ is a constant.}$$

Given that there were 360 ants when the study started,

a. show that $k = \frac{2}{3}$.

(3)

b. Show that $P = \frac{1800}{2+3e^{-0.2t}}$.

(1)

The model predicts an upper limit to the number of ants on the island.

c. State the value of this limit.

(1)

d. Find the value of t when $P = 520$. Give your answer to one decimal place.

(4)

e. i. Show that the rate of growth, $\frac{\mathrm{d}P}{dt} = \frac{P(900-P)}{4500}$

ii. Hence state the value of P at which the rate of growth is a maximum.

(5)

(Total for Question 14 is 14 marks)